# Learning mathematical proofs

## Objectives

Mathematical proofs are a fundamental part of university level mathematics and pure mathematics as a whole. Before entering university, mathematics has usually been a primarily computational subject, with the theoretical side remaining mostly unexplored. This creates a situation where a significant number of first year mathematics students feel unprepared when being introduced to university level mathematics.

The aim of this project is to create a learning platform that can aid the transition from high school to university level mathematics, particularly one that is interactive and more “hands-on” than a presentation or set of notes, that introduces the concept of proofs to incoming first year students.

## Overall structure

The learning platform will consist of several interactive tutorials that introduce core proof methods and topics that are necessary for constructing proofs such as formal logic.

Possible topics that could be included are:

| | |
|---|---|
| Fundamentals | • Sets<br>• Logic |
| Types of proof | • Direct proof<br>• Contrapositive proof<br>• Proof by contradiction<br>• Proof by induction<br>• Other types |

## Structure of each interactive tutorial

Each interactive presentation will have four key sections:

1. Introduction
2. Examples
3. Activities
4. Further questions

A brief example is on the next page.

## Direct Proofs — Section 1: Introduction

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- Direct proofs are a simple way of proving propositions or theorems that have the form of a **conditional statement**.
- …

## Direct Proofs — Section 2: Example

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Statement: If $x$ is even, then $x^2$ is even

We start with a statement and then we bridge the gap to the conclusion

Therefore, $x^2$ is even

## Direct Proofs — Section 2: Example

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Statement: If $x$ is even, then $x^2$ is even

Suppose $x$ is an even number

Then $x = 2a$ where $a$ is an integer, by definition of an even number

It is always advisable to use definitions that apply

Therefore, $x^2$ is even

## Direct Proofs — Section 2: Example

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Statement: If $x$ is even, then $x^2$ is even

Suppose $x$ is an even number

Then $x = 2a$ where $a$ is an integer, by definition of an even number

Thus $x^2 = 4a^2 = 2(2a^2)$

So $x^2 = 2b$ where $b = 2a^2$

It is always advisable to use definitions that apply

Therefore, $x^2$ is even

## Direct Proofs — Section 3: Activity

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Try reordering the intermediate steps in the proof

Statement: The product of two odd numbers $x$ and $y$ is odd

| |
|---|
| Thus $xy = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$ |
| Let $x = 2a + 1$ and $y = 2b + 1$, by definition of an odd number |
| So $xy = 2c + 1$ where $c = 2ab + a + b$ |
| Suppose $x$ and $y$ are odd |

Therefore, the product of $x$ and $y$ are odd.

## Direct Proofs — Section 4: Further learning

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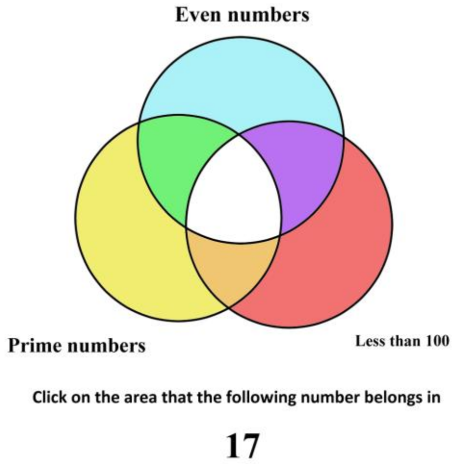
- …

# Ideas for activities




**Click on the area**

Try reordering the intermediate steps in the pr
Statement: The product of two odd numbers $x$

| |
|---|
| Thus $xy = 4ab + 2a + 2b + 1 = 2(2ab + a + b) + 1$ |
| Let $x = 2a + 1$ and $y = 2b + 1$, by definition of an odd num |
| So $xy = 2c + 1$ where $c = 2ab + a + b$ |
| Suppose $x$ and $y$ are odd |

Therefore, the product of $x$ and $y$ are odd.

**Reorder the list (drag and drop)**

$$
\begin{aligned}
& a = b \\
\implies\ & a^2 = ab \\
\implies\ & a^2 - b^2 = ab - b^2 \\
\implies\ & (a+b)(a-b) = b(a-b) \\
\implies\ & a + b = b \\
\implies\ & 2a = a \\
\implies\ & 1 = 2
\end{aligned}
$$

**Click on the incorrect step**

| $P$ | $Q$ | $P \Rightarrow Q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | |
| $F$ | $T$ | |
| $F$ | $F$ | |

**Fill in the gaps**

**Proposition** Let $a, b$ and $c$ be

*Proof.* Suppose $a \mid b$ and $b \mid c$.

Therefore $a \mid c$.

**Next step in proof (multiple choice)**

The activities are intended to create a “hands-on” experience within the tutorials. As practice and involvement are important for learning mathematics, interactive environments would provide a more engaging learning environment.